Measuring and modeling thermal fluctuations at nanometer length scales

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The size of mechanical, electrical, and optical devices continues to be reduced. As the length scales of such devices decrease, coupling to the external environment greatly increases. Thermal fluctuations due to momentum exchange between air molecules and micron scale devices under ambient conditions can effect the dynamics of a system. To illustrate this we use an atomic force microscope cantilever and detection system to measure background noise and thermal fluctuations of a micron size beam. The beam is modeled by a Langevin-type equation that is externally forced by a white-noise spectrum having an analytic as opposed to a statistical form. This model is compared with experimental data. It is found that at higher frequencies, a white-noise spectrum is not sufficient to model such a system. We modify the forcing spectrum so that it decays at higher frequencies and subsequently achieve closer agreement between the model and the experimental observations.

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I. INTRODUCTION

Recent advances in technology have made it critical that we understand the influence of thermal fluctuations in mechanical, electrical, and optical systems in much finer detail than previously required. Applications where mechanical oscillations can introduce unwanted noise in a subsequent signal are encountered in a range of devices that varies from microelectromechanical systems (MEMS) [1,2] to gravitational wave detectors [3]. In such situations, the signal of interest (e.g., capacitive reactance or interferometric fringe position) is very sensitive to the relative distance between components of the device. Thus, even though a time average of the random processes which cause all objects to vibrate about their center of mass is zero, real-time relative displacements of individual elements of the structure can become a significant source of noise. Noise is an area of concern in high-speed, high-sensitivity devices, and distinguishing the physical processes responsible for these sources of noise is necessary if we are interested in accounting for their influence.

Our goal in the present Brief Report is to characterize and determine the origins of the noise signatures in a relevant electromechanical system through mathematical models suggested by experimental data. To achieve this goal, we use a commercial atomic force microscope (AFM) that has been modified in house to allow for high-speed, real-time data collection. We monitor the noise in the system as a flexible AFM cantilever vibrates in air far from other objects, and then repeat the experiment with a rigid mirror in place of the normal cantilever. Power spectral analysis of these two data sets enables us to identify and therefore separate the noise inherent in the AFM system from that due to the thermal vibrations of the cantilever. The oscillatory motion of the cantilever is then modeled and numerically simulated by an equation of motion that includes dissipation and forcing terms to elucidate the factors controlling thermal oscillations at the nanoscale. In particular, we find that at higher frequencies a white-noise spectrum for the forcing is not sufficient to model the experimentally measured motion of the cantilever. We propose that the forcing spectrum should be modified to decay at higher frequencies, achieving closer agreement between the model and experimental observations.

II. EXPERIMENTAL METHOD

Gold-coated silicon nitride cantilevers with a nominally rectangular geometry (200 μm long, 20 μm wide) are mounted in an AFM. The vertical position of the end of the cantilever (fluctuation amplitude ~ 0.5 nm) is detected using a photodiode in the standard optical deflection method. This photodiode signal is recorded with an external data acquisition system through a modification of the control electronics. An analog to digital converter is used at a 300 kHz sampling rate to record the signal $x_i(t)$ in 20 time intervals, $(i=1,\ldots,20)$. Each signal $x_i(t)$ corresponds to 32 768 data points. To remove the influence of thermal drift on the equilibrium position of the cantilever, the signal before each interval is adjusted so that it averages to zero.

The amplitudes corresponding to each data set $x_i(t)$ are calculated by determining the absolute value of the Fourier transform $\tilde{x}_i(\omega)$ of the data. A Hanning window is used before transforming to guarantee convergence of endpoints, and then the amplitudes $\tilde{x}_i(\omega)$ are averaged. The resulting

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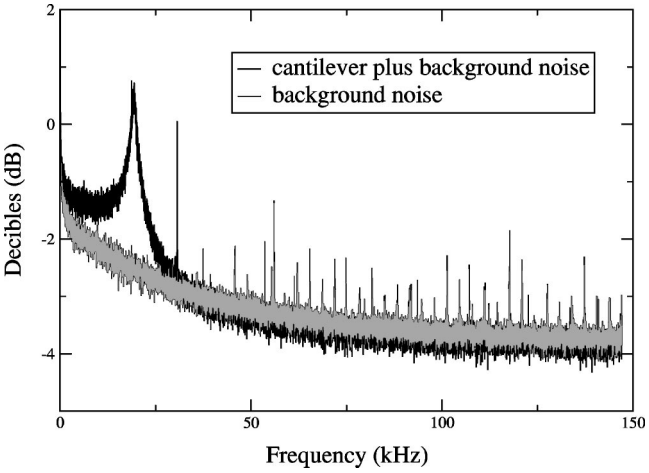

FIG. 1. Measured noise spectra.

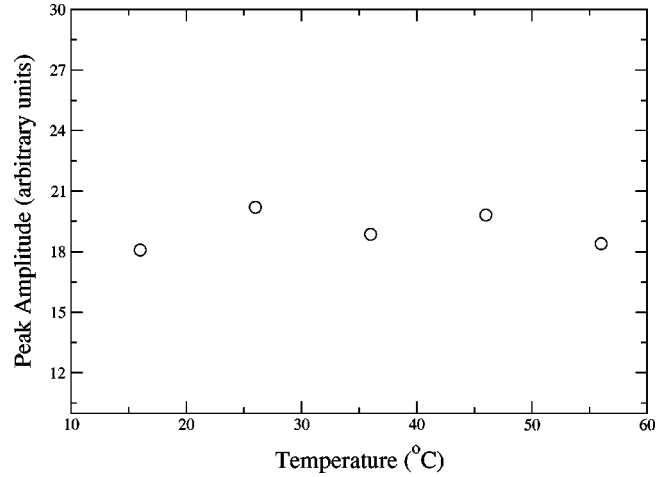

FIG. 2. Peak amplitude vs temperature.

averaged amplitude $\tilde{x}(\omega)$ represents that of the system including the cantilever. Next, a fixed mirror tilted at roughly the same angle as that of the cantilever is used in its place, and the same sampling and averaging procedure is used to measure the background noise amplitude $\tilde{n}(\omega)$ as for the cantilever amplitude $\tilde{x}(\omega)$. The cantilever thermal noise amplitude $\tilde{X}(\omega)$ is then calculated by computing the difference, $\tilde{X}(\omega)=\tilde{x}(\omega)-\tilde{n}(\omega)$.

III. RESULTS

A. Extracting Cantilever Noise

Figure 1 presents data resulting from the procedures described above, namely the averaged amplitudes $\tilde{x}(\omega)$ and $\tilde{n}(\omega)$. Their difference $\tilde{X}(\omega)$ is shown as the data in Figs. 3 and 5. It can be easily seen in Fig. 3 that the procedure of subtracting out the noise inherent in the system due to the electronics, fluctuations in the laser and photodiode, extraneous mechanical oscillations, etc., yield a spectrum that is very well defined. Thermal noise has been an issue in AFM technology that has received significant attention [4–12]. The present results show that amplitude averaging and subtraction can be used to elucidate only those noise signatures associated with the cantilever motion. It should be noted that these results are for the cantilever in ambient air, and that for this particular cantilever only one vibrational mode is detected. Some cantilevers we have tested exhibit more than one mode, but we concentrate here on the simplest type of cantilever as a proof of concept for our data acquisition and analysis approach.

An interesting and possibly counterintuitive result is presented in Fig. 2, where the peak amplitude of the cantilever is plotted versus temperature. Over the limited temperature range studied the cantilever amplitude is essentially constant, indicating a lack of sensitivity to the local thermal environment. The temperature measurements are performed with a thermocouple suspended very close to the end of the cantilever and reasonably uniform heating and cooling of the air in close proximity to the cantilever is accomplished by placing the entire AFM head on a hot plate or on ice, respectively. This rather crude heating/cooling technique limits our temperature range but the data clearly demonstrate that local temperature is not a dominant factor in the amplitude of oscillation of the lever arm.

The lack of temperature dependence involves the viscosity of the air that acts to damp the vibration of the cantilever in addition to its inherent damping mechanisms. Viscosity is an energy/momentum-transfer process between, in this case, a solid object and a compressible fluid. Detailed analysis for air [7] indicates that the viscosity varies linearly with absolute temperature. However, the expected increase in damping (and subsequent decrease in amplitude) may be balanced by the increase in thermal energy supplied to the cantilever (and subsequent increase in amplitude), resulting in an insensitivity of the amplitudes to temperature as shown in Fig. 2.

B. Modeling Cantilever Noise

The governing equation for the cantilever deflection is obtained by assuming that we have a clamped cantilever resting on an elastic foundation. In the limit of small flexural rigidity ($EI \ll 1$, where E is the modulus of elasticity and I the moment of inertia) this reduces to the familiar equation of motion for a one-dimensional damped, driven oscillator

$$\ddot{y}(t)+2\omega_0 a\dot{y}(t)+\omega_0^2 y(t)=F(t), \tag{1}$$

where $y(t)$ represents the location of the end of the cantilever, a is the intrinsic damping factor, ω_0 is the resonant angular frequency, and $F(t)$ is an external forcing term. The constant a represents the damping coefficient associated with linear mechanisms. We consider the forcing term $F(t)$ to be due to random thermal processes. In our numerical simulations we represent this term using the form presented by Shinozuka [13]

$$F(t)=f\sqrt{\frac{A^2}{N}}\sum_{n=1}^{N}\cos(2\pi\nu_n t+\phi_n). \tag{2}$$

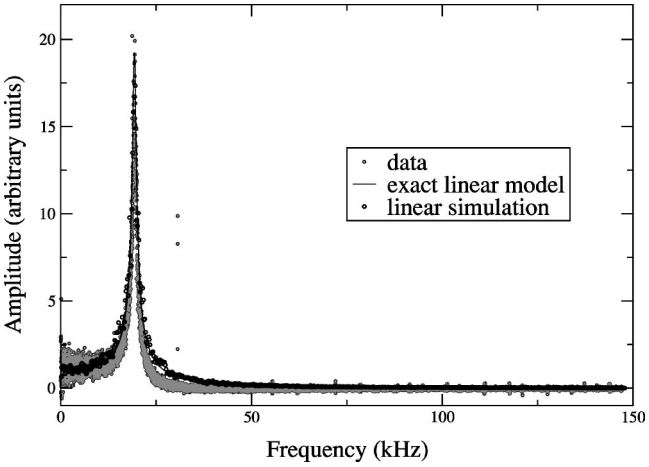

FIG. 3. Linear damping simulation: $\omega_0/(2\pi)=19.29$ kHz, $a=0.042$, and $f=1.56$.

This form is chosen to represent a white-noise spectrum. The frequencies (ν_n) are random numbers varying between 0 and 150 kHz, where 150 kHz is one half the data collection rate. The relative phases (ϕ_n) are random phases between 0 and 2π. In this case, we can exactly solve Eq. (1) by Fourier transformation. We also solve Eq. (1) numerically using Eq. (2) for the external forcing with $N=1000$. The numerical simulation is conducted 20 times to generate the model signal $y_i(t)$. These signals are then Fourier transformed and averaged as are the experimental data $x_i(t)$. We present the results in Fig. 3, where the parameters a and f have been chosen by nonlinear regression. Here, we compare the experimental data, the exact solution obtained by Fourier transformation, and the numerical solution. This comparison between the exact solution and the numerical solution verifies that our numerical simulation procedure closely approximates the exact solution. The comparison with experimental data demonstrates that the linear model reproduces physically salient features of the system.

We also compare the data against simulations using an additional power-law dissipation term, of the form $b(t)\dot{y}^3(t)$. However, for a wide variety of parametric

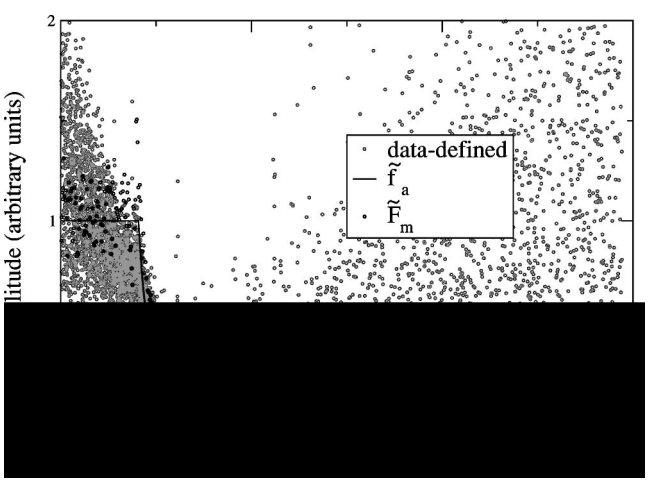

FIG. 4. Forcing spectrum $\tilde{f}(\omega)$.

FIG. 5. Modified forcing simulation.

choices, the numerical results are all but indistinguishable from those obtained from the system with linear damping alone. We conclude that nonlinear damping terms alone are insufficient to account for the discrepancies seen between the experimental data and the mathematical model at high frequency (>25 kHz).

IV. DISCUSSION

Figure 3 demonstrates that we are able to achieve a reasonable fit to the data at and below the resonant peak. Since Eq. (1) is by far easier to implement than nonlinear models, and admits an exact solution, the present Brief Report demonstrates why the linear model subject to white-noise forcing is commonly used in the literature to fit the peak region for calibration purposes.

However, we note common discrepancies between the experimental data and the results from the analytical modeling. Although the low-frequency response and resonant peak are accurately represented by the simulations, the response above the resonant peak is systematically over predicted by the analytical models. One possible explanation arises from the external forcing. In the previous analysis it has been assumed that the noise was white, so that all frequencies are equally represented. However, there is no reason to expect that all frequencies have identical amplitudes. If the noise is not white, and instead has some structure, this could account for errors seen in the analytical predictions. Such cases are discussed at length in [14]. It is clear that in some situations there may be a critical time constant, or in our case a critical frequency, at which a white-noise forcing spectrum must be replaced by a colored spectrum in order to accurately model reality.

To illustrate we consider the Fourier transform of Eq. (1)

$$\tilde{Y}(\omega)=A_0\tilde{f}(\omega)[(\omega_0^2-\omega^2)^2+(2a\omega_0\omega)^2]^{-1/2}, \qquad (3)$$

where $\tilde{Y}(\omega)$ is the transform amplitude of $y(t)$ and $A_0\tilde{f}(\omega)$ is the transform amplitude of the external forcing $F(t)$. White noise assumes that $\tilde{f}(\omega)=1$ so that A_0 is the constant amplitude of the white-noise forcing. The case $\tilde{f}(\omega)=1$ is the exact linear model referred to in Fig. 3.